Final exam

Electronics & Signal processing
06-04-2016

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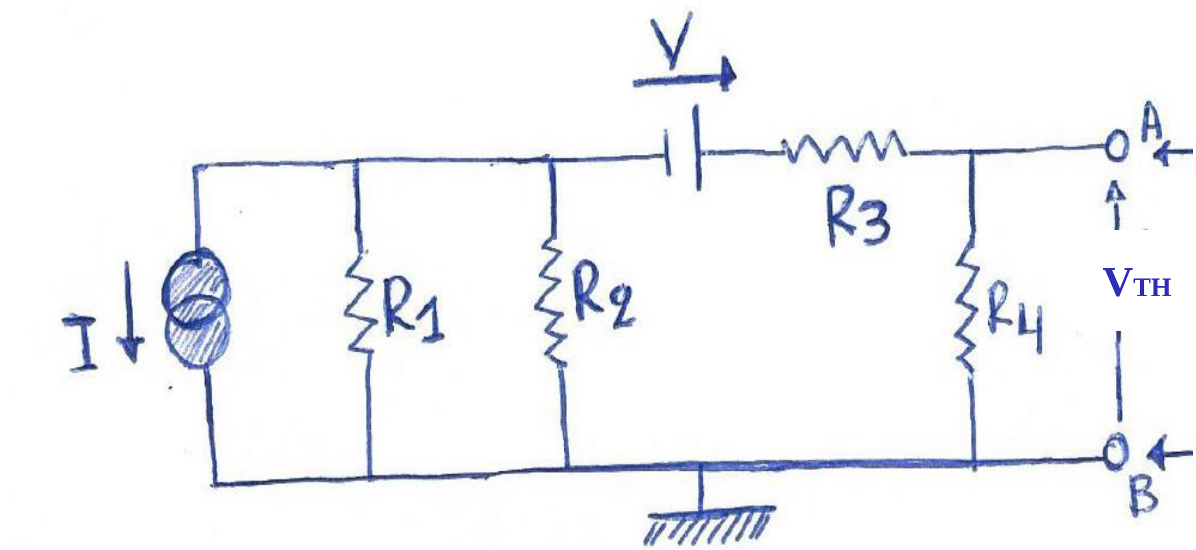
Grade of written exam:

Mark is cummulative points scored for all problems

Total maximum score : 10

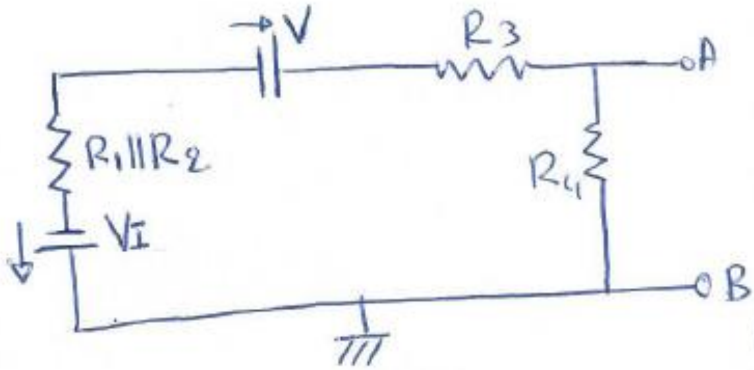
Problem 1 (1.5 points)



Derive the Thévenin equivalent between points A and B by calculating the Thévenin potential $\mathbf{V_{TH}}$ (1 point) and the Thévenin resistance $\mathbf{R_{TH}}$ (0.5 points) using only the Thevenin-Norton concepts to analyze the circuit.

Solution Problem 1

Transform first to have only voltage sources

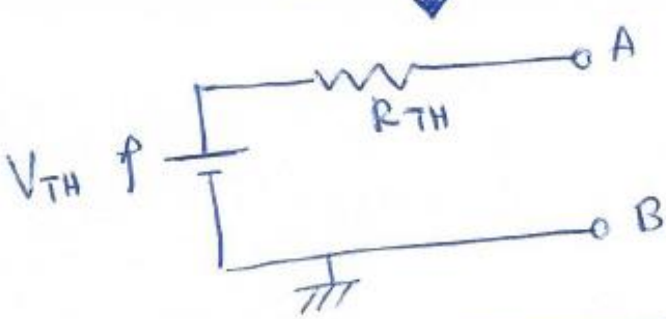


$$V_I = I \cdot (R_1 || R_2)$$

$$\left\{ R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} \right.$$ Resisters in parallel $\}$

$$V_S = V - I(R_1 || R_2)$$

$$R_S = R_3 + R_1 || R_2$$



$$V_{TH} = V_S \frac{R_4}{R_S + R_4} \Rightarrow$$

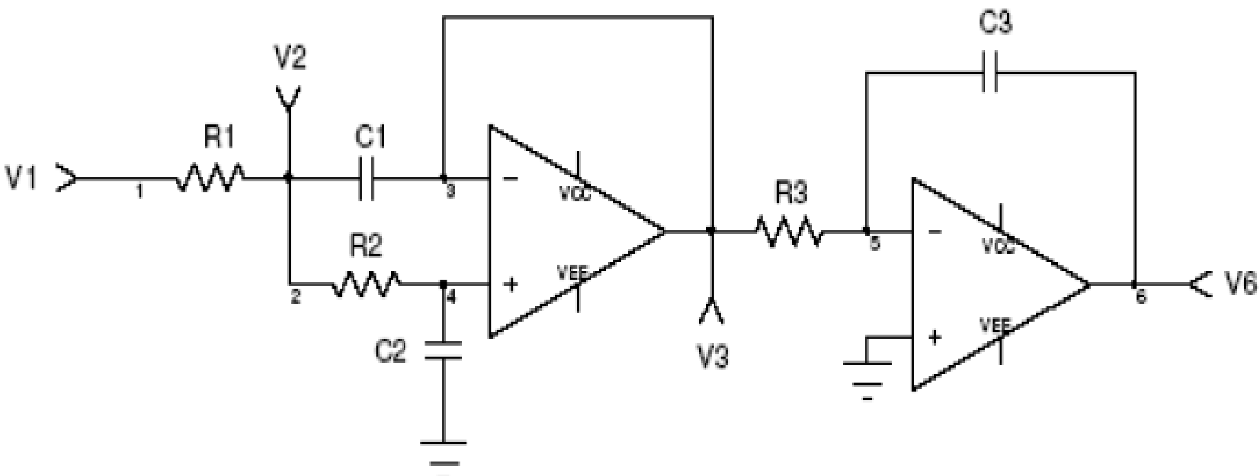
$$V_{TH} = \left[V - I(R_1 || R_2)\right] \frac{R_4}{R_S + R_4}$$

$$\Rightarrow \boxed{V_{TH} = \left[V - I(R_1 || R_2)\right] \frac{R_4}{R_3 + R_4 + (R_1 || R_2)}}$$ (1 point)

$$\boxed{R_{TH} = R_S || R_4 = (R_3 + R_1 || R_2) || R_4}$$ (0.5 points)

Problem 2 (2.5 points)

Consider the circuit below with two ideal op amps (**V+=V-**)



(a: 0.5 points) Show that : $\frac{V(6)}{V(3)} = -\frac{1}{j\omega\tau_3}$

where $\tau_3 = R_3C_3$.

(b: 1.5 points) Show that

$$V(2) = \frac{(\frac{R_1}{R_2} + j\omega\tau_1)\,V(3) + V(1)}{1 + \frac{R_1}{R_2} + j\omega\tau_1}$$

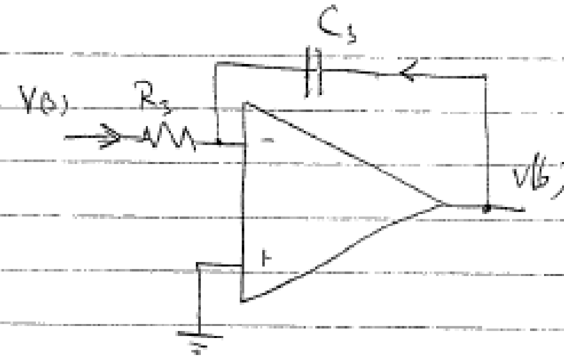
where $\tau_1 = R_1C_1$.

(c: 0.5 points) Show that $V(3) = V(4) = \frac{V(2)}{1 + j\omega\tau_2}$

where $\tau_2 = R_2C_2$.

Solution Problem 2

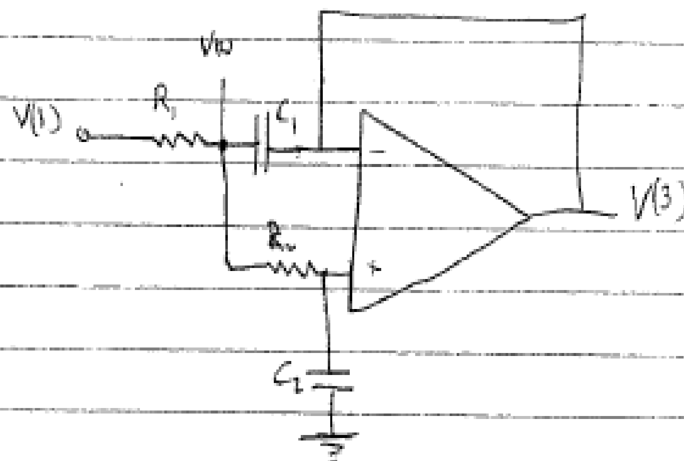
(a)



a) $$\frac{V(3)}{R_3} + \frac{V(6)}{\frac{1}{j\omega C_3}} = 0$$

$$\frac{V(6)}{V(3)} = -\frac{1}{j\omega R_3 C_3} = -\frac{1}{j\omega \tau_3}$$

(b)



$$V(2) = \frac{\frac{1}{j\omega C_1} \,//\, R_2}{R_1 + \frac{1}{j\omega C_1} \,//\, R_2} V(1) + \frac{R_1}{R_1 + \frac{1}{j\omega C_1} \,//\, R_2} V(3)$$ (1 point)

$$\frac{1}{j\omega C_1} \,//\, R_2 = \frac{\frac{R_2}{j\omega C_1}}{R_2 + \frac{1}{j\omega C_1}} = \frac{R_2}{1 + j\omega R_2 C_1}$$

$$R_1 + \frac{1}{j\omega C_1} // R_2 = R_1 + \frac{R_2}{1 + j\omega R_2 C_1} = \frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{1 + j\omega R_2 C_1}$$

$$\therefore V(2) = \frac{R_2}{R_1 + R_2 + j\omega R_2 R_1 C_1} V(1) + \frac{R_1 + j\omega R_2 R_1 C_1}{R_1 + R_2 + j\omega R_2 R_1 C_1} V(3)$$

$$V(2) = \frac{1 \quad V(1)}{1 + \frac{R_1}{R_2} + j\omega \tau_1} + \frac{\frac{R_1}{R_2} + j\omega \tau_1}{1 + \frac{R_1}{R_2} + j\omega \tau_1} V(3)$$

$$V(2) = \frac{\left(\frac{R_1}{R_2} + j\omega \tau_1\right) V(3) + V(1)}{1 + \frac{R_1}{R_2} + j\omega \tau_1}$$ (0.5 points)

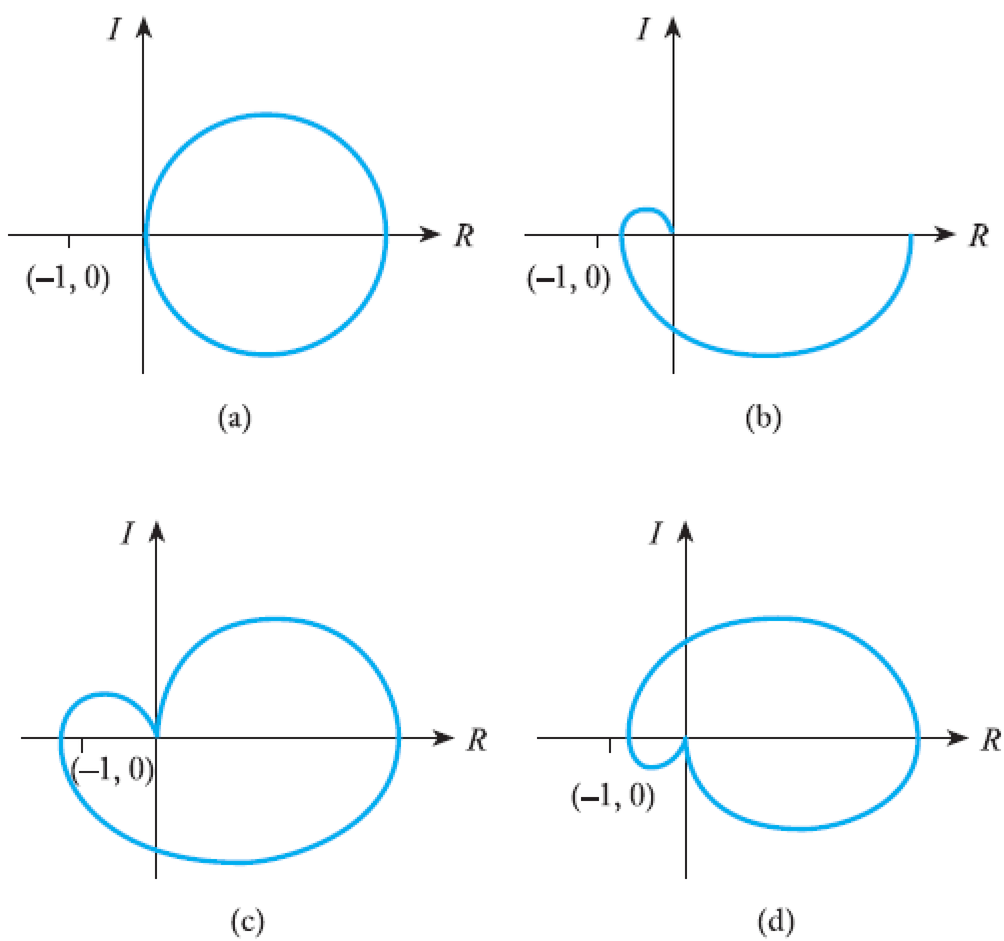
c) $$V(4) = \frac{\frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} V(2) = \frac{1}{1 + j\omega R_2 C_2} V(2)$$

$$\therefore V(3) = V(4) = \frac{1}{1 + j\omega \tau_2} V(2) \qquad \tau_2 = R_2 C_2$$

Problem 3 (1 point)

The Nyquist diagrams below represent four circuits (0.25 points per circuit). In each case determine:

- the number of low and high-frequency cut-offs
- whether or not the circuit is stable.



Solution Problem 3

1a) 1 high cut-off
1 low cut-off
stable

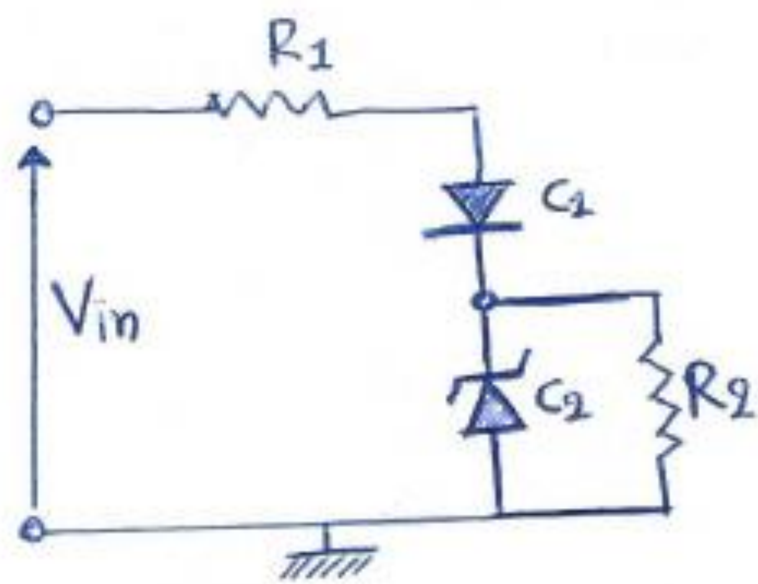
b) 3 high cut-offs
stable

c) 3 high cut-offs
1 low cut-off
unstable

d) 1 high cut-off
3 low cut-offs
stable

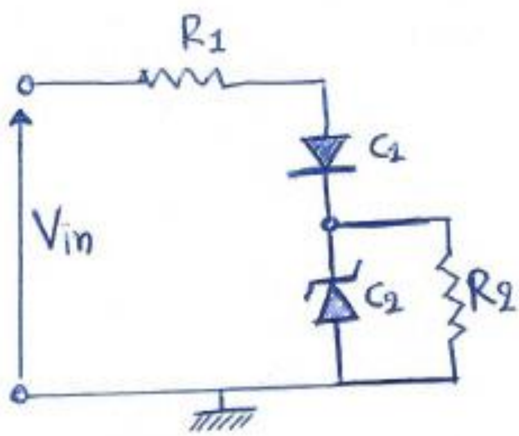
Problem 4 (1.5 points)

The diodes C_1 and C_2 are assumed ideal with forward conduction voltage $V_c \ll V_{in}$ (>0). The Diode C_2 is also a Zener diode with reverse conduction voltage V_z ($\ll V_{in}$).



Calculate the current through the resistor R_2 and discuss possible restrictions for the resistor ratio $R1/R2$ depending on the operation of C1 and C2.

Solution Problem 4



$V_{in} >> V_c$ → Diode C1 conducts
C2 can conduct only as Zener

If we ignore zener C2 then the potential at point "O" is

$$V_o = V_{in} - V_c - I R_1 \quad (1)$$

$$I = \frac{V_{in} - V_c}{R_1 + R_2} \quad (2)$$

(1) & (2) give

$$\boxed{V_o = (V_{in} - V_c)\frac{R_2}{R_1 + R_2}} \quad (3)$$

(I) If $V_o < V_z$ the diode C_2 is not conducting as a zener ⇒

$$\boxed{I_2 = I = \frac{V_{in} - V_c}{R_1 + R_2}} \quad (4)$$

(0.5 points)

(II) If $V_o \geq V_z$ then the zener is conducting and the potential at point o is V_z =>

$$I_2 = \frac{V_z}{R_2} \quad (5)$$ (0.5 points)

since $V_o \geq V_z$ => $(V_{in} - V_c)\frac{R_2}{R_1+R_2} \geq V_z$ =>

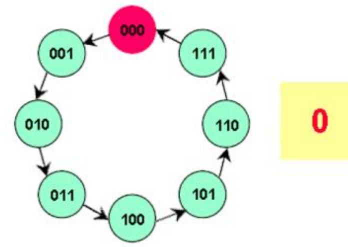
(0.5 points)

$$\frac{R_1}{R_2} + 1 \leq \frac{V_{in} - V_c}{V_z} \Rightarrow \boxed{\frac{R_1}{R_2} \leq \frac{V_{in} - V_c}{V_z} - 1} \quad (6)$$

Restriction for R_1/R_2

If (6) is not valid then the zener is not conducting!

Problem 5 (2 points)



(a: 1 point) Using the output table given below, design the synchronous 8-counter 0→7 (X: do not care) using 3 J-K flip-flops:

Output State Transitions		Flip-flop inputs		
Present State Q2 Q1 Q0	Next State Q2 Q1 Q0	J2 K2	J1 K1	J0 K0
0 0 0	0 0 1	0 X	0 X	1 X
0 0 1	0 1 0	0 X	1 X	X 1
0 1 0	0 1 1	0 X	X 0	1 X
0 1 1	1 0 0	1 X	X 1	X 1
1 0 0	1 0 1	X 0	0 X	1 X
1 0 1	1 1 0	X 0	1 X	X 1
1 1 0	1 1 1	X 0	X 0	1 X
1 1 1	0 0 0	X 1	X 1	X 1

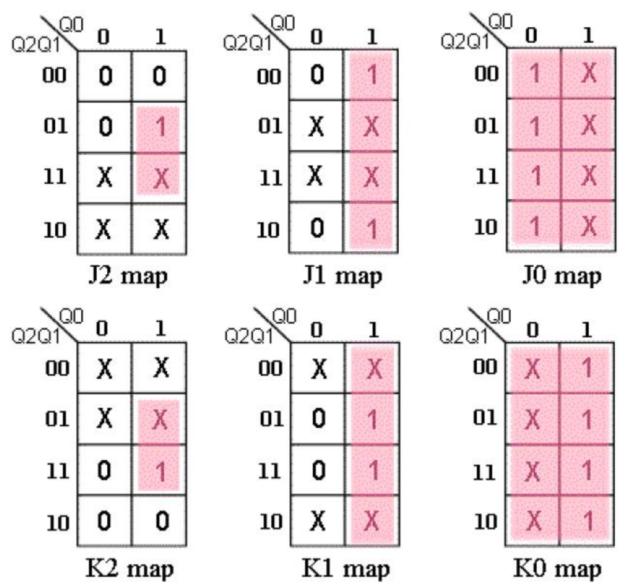
(b: 1 point) Using the result from (a) identify J-K flip-flops acting as internal clocks in order to design a simpler version of the synchronous 8-counter 0→7 (asynchronous)

(a)

Solution Problem 5

Output State Transitions		Flip-flop inputs		
Present State Q2 Q1 Q0	Next State Q2 Q1 Q0	J2 K2	J1 K1	J0 K0
0 0 0	0 0 1	0 X	0 X	1 X
0 0 1	0 1 0	0 X	1 X	X 1
0 1 0	0 1 1	0 X	X 0	1 X
0 1 1	1 0 0	1 X	X 1	X 1
1 0 0	1 0 1	X 0	0 X	1 X
1 0 1	1 1 0	X 0	1 X	X 1
1 1 0	1 1 1	X 0	X 0	1 X
1 1 1	0 0 0	X 1	X 1	X 1

The K-maps for J-Ks are:

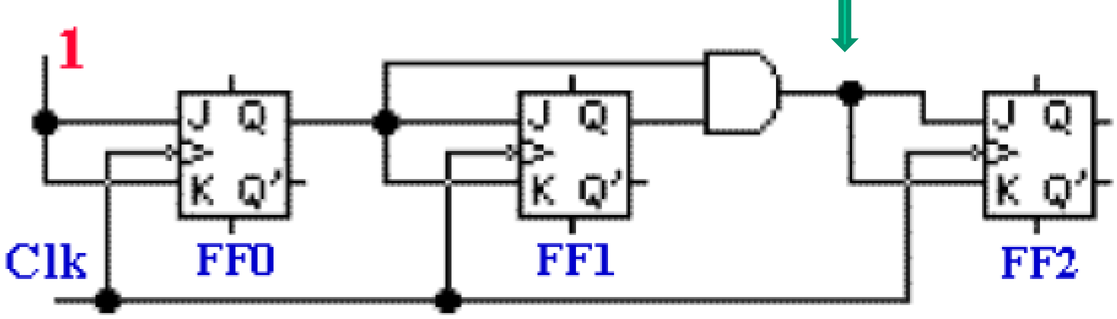


From the K-maps we obtain after crouping: →

J0 = K0 = 1

J1 = K1 = Q0

J2 = K2 = Q1*Q0



(b)

Output State Transitions		Flip-flop inputs		
Present State Q2 Q1 Q0	Next State Q2 Q1 Q0	J2 K2	J1 K1	J0 K0
0 0 0	0 1	0 X	~~0~~ **X** X	1 X
0 0 1	1 0	0 X	1 X	X 1
0 1 0	1 1	0 X	X ~~0~~ **X**	1 X
0 1 1	0 0	1 X	X 1	X 1
1 0 0	0 1	X 0	~~0~~ **X** X	1 X
1 0 1	1 0	X 0	1 X	X 1
1 1 0	1 1	X 0	X ~~0~~ **X**	1 X
1 1 1	0 0	X 1	X 1	X 1

(0.5 points)

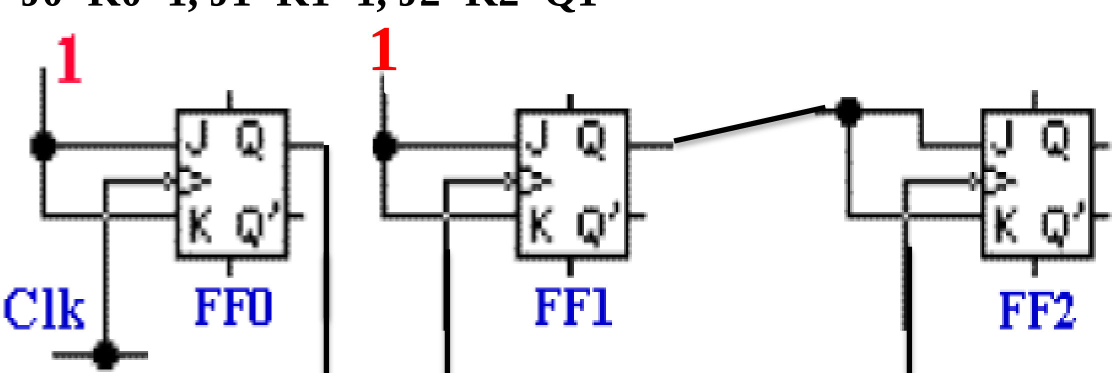
Q0 clock for Q1 (Q1 changes only when Q0 has negative drop→ Extra do not cares **X** for J1 and K1 ⇒ **J1=K1=1**

Output State Transitions		Flip-flop inputs		
Present State Q2 Q1 Q0	Next State Q2 Q1 Q0	J2 K2	J1 K1	J0 K0
0 0 0	0 1	~~0~~ **X** X	0 X	1 X
0 0 1	0 0	0 X	1 X	X 1
0 1 0	0 1	~~0~~ **X** X	X 0	1 X
0 1 1	1 0	1 X	X 1	X 1
1 0 0	1 1	X ~~0~~ **X**	0 X	1 X
1 0 1	1 0	X 0	1 X	X 1
1 1 0	1 1	X ~~0~~ **X**	X 0	1 X
1 1 1	0 0	X 1	X 1	X 1

(0.5 points)

Q0 clock for Q2 → Extra do not cares **X** as indicated. If we derive the K-maps then we obtain 1⇒ **J2=K2=Q1**

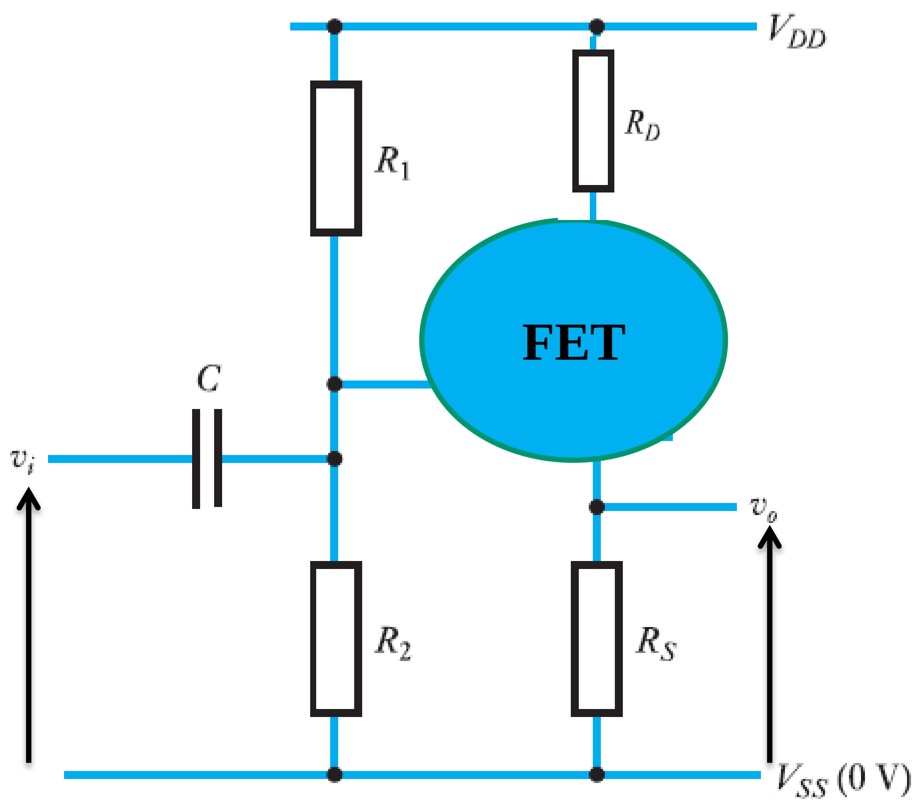
J0=K0=1, J1=K1=1, J2=K2=Q1



You can also consider Q1 clock for Q2 and then: J1=K1=J2=K2=J0=K0=1

Problem 6 (1.5 points)

Consider a FET amplifer as it is shown bellow:



Show that the amplification ratio of the input/output potential variations υo/υi is given by:

$$\frac{\upsilon_o}{\upsilon_i} = \frac{g_m R_s}{1 + g_m R_S + [(R_D + R_S)/r_d]}$$

with g_m the transconductance and r_d the differential resistance of the FET operating at saturation.

Solution Problem 6

First method: Solution for tough cookies!

I use differential analysis. [you can also use small signal circuit

(1) $V_o = I_D \cdot R_S$, $\boxed{U_s \equiv U_o}$ $\boxed{U_g \equiv U_i}$

(2) $\tilde{V}_o = \tilde{I}_D R_S$, $\tilde{I}_D = I_D + g_m U_{gs} + \frac{U_d - U_s}{r_d}$,

change of FET current because of U_i

$U_s = \tilde{V}_o - V_o (\equiv U_o)$

(2) − (1) ⇒ $U_s = R_S \left[g_m U_{gs} + \frac{U_d - U_s}{r_d} \right]$ (3)

(4) $I_D = \frac{V_S}{R_S} = \frac{V_{DD} - V_D}{R_D}$, $\tilde{I}_D = \frac{\tilde{V}_S}{R_S} = \frac{V_{DD} - \tilde{V}_D}{R_D}$ (5)

(5) − (4) ⇒ $\frac{U_s}{R_S} = -\frac{U_d}{R_D}$ (6)

(6) ⇒ $U_d = -\frac{R_D}{R_S} U_s$ and substitute in (3)

$$U_s = g_m R_S (U_g - U_s) - R_S \frac{1 + \frac{R_D}{R_S}}{r_d} U_s$$

$$U_s = g_m R_S U_g - g_m R_S U_s - \frac{R_S + R_D}{r_d} U_s \Rightarrow$$

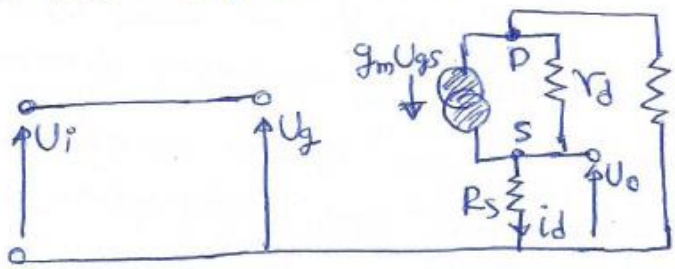
$$U_s \left[1 + g_m R_S + \frac{R_S + R_D}{r_d} \right] = g_m R_S U_g$$

$U_s \equiv U_o$, $U_i \equiv U_g$

$$\frac{U_o}{U_i} = \frac{g_m R_S}{1 + g_m R_S + \frac{R_D + R_S}{r_d}}$$

Second method: Solution for normal cookies!

Small signal circuit



(1) $U_o = U_s = i_d R_s$, $i_b = g_m U_{gs} + \frac{U_d - V_s}{r_d}$ (2)

$i_b = -\frac{U_d}{R_D} = \frac{U_s}{R_S} \Rightarrow U_d = -\frac{R_D}{R_S} U_s$ (3)

(1) & (2) & (3) $\Rightarrow$

$$U_s = R_s\left[g_m U_g - g_m U_s - \frac{1 + \frac{R_D}{R_S}}{r_d} U_s\right]$$

$$\Rightarrow U_s\left[1 + g_m R_s + \frac{R_D + R_S}{r_d}\right] = g_m R_s U_g$$

$U_s = U_o$, $U_i = V_o$

so we obtain

$$\frac{U_o}{U_i} = \frac{g_m R_s}{1 + R_s g_m + \frac{R_D + R_S}{r_d}}$$